Simulations of the Behavior of Helium in a Thermal Conductivity Cell

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We have performed numerical simulations of the behavior in a one-dimensional thermal conductivity cell of helium as it is ramped through the superfluid transition from below. The goal of the simulations is to be able to at least qualitatively, if not quantitatively, predict the behavior of helium in a reduced gravitational environment. These numerical simulations can model helium cells with effective gravitational fields of $|a/g| > 0$ by changing the distribution of local transition temperatures to match the desired gravitational environment. The numerical results for the simulated behavior show excellent qualitative agreement with the observed experimental data under a variety of effective gravity values.

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1. INTRODUCTION

When a side temperature probe with $\sim 10^{-9}K$ resolution was first attached to a thermal conductivity cell in the late 1980's,[1] several previously unexpected phenomena were observed because of the high temperature resolution thermometry and the unique view point. Those data were taken by applying a constant heat current to the bottom of the thermal conductivity cell, ramping the temperature of the top of the cell at a constant rate starting from below the transition temperature for the bottom of the cell ($T_\lambda(z = 0)$), and ramping until the whole cell was significantly above the transition. When the temperature at the middle of the cell was monitored, it was discovered that the helium ramp rate dropped below the ramp rate of the top of the cell once normal fluid appeared in the cell ($T > T_\lambda(z = 0)$). It also was observed that the temperature rise at the middle of the cell, once

$T_{He} > T_\lambda(z = 0.5L)$, was noticeably slower than the temperature rise seen at the bottom of the cell when normal fluid first entered the cell. Because of these experimental observations, there developed an interest to numerically model the behavior of a thermal conductivity cell of liquid helium-4 ramped through the superfluid transition. Therefore, a simple one dimensional model was developed,[2] and this model was used with some success to provide a bit of insight into these experimentally observed phenomena.

More recently there has developed a strong interest in the effects of gravity on the λ-transition. In traditional ground based experiments there are gravitationally induced pressure variations in any macroscopic helium sample that limit how closely the transition can be approached. Currently there are several experimental efforts underway to study the thermal conductivity near the λ-transition in a reduced gravitational environment, by either performing the experiments in low earth orbit or by utilizing the low-g simulator at JPL.[3] Because of these experimental efforts, there is a renewed interest in being able to model the behavior of a helium thermal conductivity cell, but with an emphasis on modeling the behavior in a reduced gravitation environment. So, the original numerical model has recently been expanded to include the ability to vary the acceleration due to gravity acting on the thermal conductivity cell.

Below we describe the numerical technique used in our model. The model simluates the experimental thermal conductivity cell by using the experimentall measured values for the Kapitza resistance, the applied heat current, and the ramp rate. The results of the model are described, and a comparison of the numerical results with experimental data is presented. Finally, we will discuss some of our plans for extending this model.

2. NUMERICAL TECHNIQUE

The problem we are interested in simulating is that of a thermal conductivity cell of helium initially below the λ-transition temperature. The top temperature of the cell is ramped in time at a constant rate while a constant heat current, Q_{bot}, is applied from the bottom of the cell. The transition temperature will be height dependent due to the pressure dependence of the transition, so the bottom of the cell will pass through the transition first. While the cell remains below the transition temperature of the bottom of the cell, all of the helium in the cell will be essentially isothermal due to the effectively infinite thermal conductivity of the superfluid helium. Also, the helium in the cell will always be at a higher temperature than the copper at the top of the cell due to the Kapitza resistance across the top boundary of the cell. When the helium temperature reaches T_λ for the bottom of the

cell, a temperature gradient will develop across the normal fluid portion of the cell. The goal of the model is to be able to calculate the temperature profile of the cell as a function of time for a given bottom heat current and top temperature ramp rate. This calculated temperature profile can then be used to find the temporal evolution of the temperature at any point in the cell, and it also can be used to find the amount of heat that passes through the cell, $Q_{out} \leq Q_{bot}$.

To numerically model the cell, we start with the heat continuity equations. These equations can be combined in the case where the thermal conductivity is position dependent to give the following equation:

$$\frac{dT}{dt} = (\lambda/\rho C)\nabla^2 T + \frac{\nabla\lambda\nabla T}{\rho C} \tag{1}$$

where ρ is the density of the helium, C is the specific heat of the helium, λ is the thermal conductivity of the helium, and T is the temperature of the helium. Because the thermal properties of helium near the λ-transition are functions of the reduced temperature $\epsilon \equiv T/T_{\lambda(z)} - 1$, it is necessary to transform equation 1 into a function of the reduced temperature. To accomplish this transformation, one must include the spatial dependence of the transition temperature:

$$T_\lambda(z) = T_\lambda(0) + z(a)\rho(dT/dP)_\lambda \tag{2}$$

where z is the distance above the bottom of the cell, a is the acceleration (gravity) acting on the cell, and $(dT/dP)_\lambda$ is the pressure dependence of the λ-transition. In one g and at saturated vapor pressure, the coefficient for z in equation 2 is $\nabla T_\lambda = 1.273 \times 10^{-6}(K/cm)$. After taking all appropriate derivatives and making the necessary substitutions,[2] equation 1 can be rewritten as:

$$\frac{d\epsilon}{dt} = D\{\nabla^2\epsilon + (\nabla\epsilon)\frac{\nabla T_\lambda}{T_\lambda}[2 + \frac{d(ln(\lambda))}{d\epsilon}(1+\epsilon)] + (\nabla\epsilon)^2(\frac{d(ln(\lambda))}{d\epsilon})\} \tag{3}$$

where $D = \lambda/(\rho C)$ is the diffusivity of the helium.

With the differential equation transformed into the physically relevant variable, the equation can then be discretized using a 2-level Crank Nicholson implicit scheme.[4] This scheme averages the right hand of equation 3 at the time steps n and $n+1$, and will be second order accurate in time. When this method is applied to equation 3, one must be careful to keep the equation first order in terms evaluated at the later time $(n+1)$. So, it is assumed that ϵ does not change much between consecutive time steps.

Once all the appropriate approximations are applied, equation 3 can be written as a discrete equation for a given time step n as an array tridiagonal

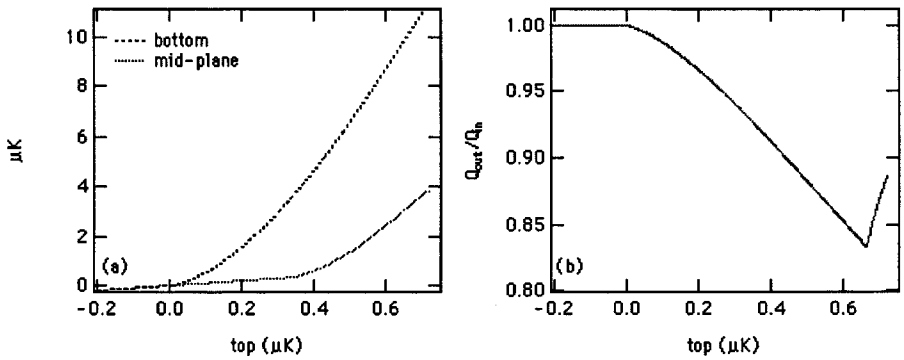

Fig. 1. Numerical simulation a 0.5cm long cell with $Q_{bot} = 0.1\mu W/cm^2$, a ramp rate of $1.1nK/sec$, and a Kapitza resistance of $1.69Kcm^2/W$. (a) The temperature of the bottom and the middle of the cell as a function of the temperature of the top of the cell. (b) The ratio of Q_{out} and Q_{bot} as a function of the temperature of the top of the cell.

in ϵ_j^{n+1}, where j refers to the spatial grid position. This matrix can then be solved using a tridiagonal matrix inversion routine.[4] The coefficients for the ends of the spatial grid are found from the applied boundary conditions: constant heat inputted at $z = 0$, and controlled temperature at $z = L$. Also, since the superfluid portion of the cell will be isothermal with the helium at the top of the cell, the code is written so that it only solves for the temperature profile in the portion of the cell that contains normal fluid. This gives the model an effectively varying cell length as the superfluid/normalfluid interface moves through the cell.

To match the simulations with the physical cells, the top boundary condition includes the temperature drop between the cell helium and the controlled copper end cap. Numerically this was accomplished by calculating at each time step how much of the heat applied to the bottom of the cell, Q_{bot}, is lost as it is used to warm the normal fluid portion of the cell. Once the heat leaving the normal fluid, Q_{out}, is known, the temperature drop across the top boundary of the cell is calculated using the Kapitza resistance of that boundary.

Finally, the model was modified to include a user specified value for the acceleration level acting on the helium in the cell. This modification was made by returning to equation 3, and replacing $\nabla T_\lambda = 1.273 \times 10^{-6}$ with $\nabla T_\lambda = (a/g)1.273 \times 10^{-6}$.

3. NUMERICAL RESULTS

A typical numerically simulated data set is shown in figure 1. The simulation was done with an applied heat current of $0.1\mu W/cm^2$, a ramp rate for the top copper end cap of $1.1nK/sec$, a top boundary or Kapitza resistance of $1.69Kcm^2/W$, and a cell length of $0.5cm$. As can be seen from the figure, the bottom of the cell starts to warm as soon as the top of the cell warms above $T_\lambda(z = 0)$. The heat current passing through the cell, Q_{out}, also starts to drop once normal fluid is present in the cell. Because of the low heat current and ramp rate used in this simulation, there is very little difference between the ramp rate of the copper at the top of the cell and the ramp rate of the helium, but the top of the cell has to warm $\sim 12nK$ above the transition temperature of the middle of the cell before the helium reaches that temperature. Finally, once the cell is full of normal fluid, there is a small inflection in the warming rates of the bottom and middle of the cell, and the heat coming through the cell starts to increase. This behavior is consistent with the experimentally observed behavior.

3.1. Comparison To Experimental Results

With these simulations it is interesting to study the effect of varying the various experimental parameters, and then compare the simulation's results with previously gathered experimental data. There are four relevant parameters that can be varied experimentally: the applied heat current, the top ramp rate, the effective gravity, and the top Kapitza resistance. Since the current goal of the simulations is to be able to use the model to predict the behavior in a reduced gravitational environment, we will focus on those results in this paper. Figure 2 shows the effect of varying the acceleration level in the numerical simulations and on a physical cell (figure 2). The simulation and the physical data have different applied heat currents ($Q_{bot} = 0.4$ and $0.9\mu W/cm^2$), but all other parameters were equal. The value of the heat current used in the simulation was choosen so that the simulations were matched to behavior of the physical cell. As can be seen from the figure, the qualitative behavior is very similar for the real data versus the simulations, but it does seem like the simulations predict somewhat larger changes in the behavior as the effective gravity is lowered than what is seen in the physical system.

4. CONCLUSIONS

The numerical model presented here gives good qualitative agreement with the behavior observed in physical helium conductivity cells, but there

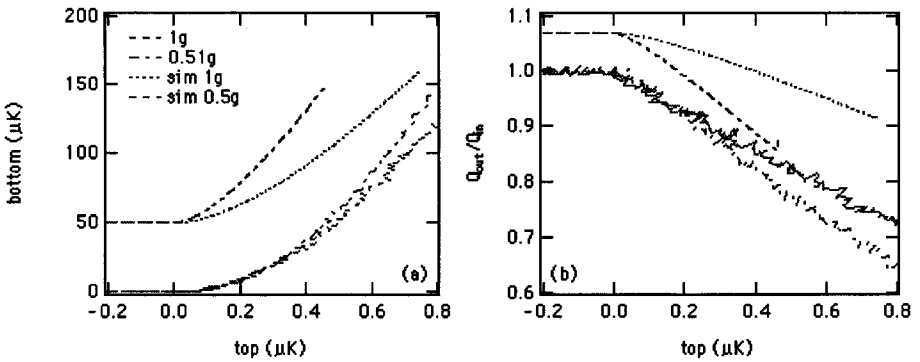

Fig. 2. Comparison of varying the acceleration level in the simulations, $Q_{bot} = 0.4\mu W/cm^2$, and in a physical cell, $Q_{bot} = 0.9\mu W/cm^2$. Ramp rate ($0.55nK/s$) and Kapitza resistance ($1.69Kcm^2/W$) were the same for the simulations and the physical cell. The simulation data has been shifted with respect to the physical data for clarity. (a) The bottom temerature as a function of the temperature of the top of the cell.(b) The ratio of Q_{out} and Q_{bot} as a function of the temperature of the top of the cell.

is a need for further refinement of the model to be able to make quantitative comparisons of the simulations and physical data. The model already has been very useful for both data analysis and as a tool for choosing appropriate experimental parameters. One of the improvements that will be added to the model is the Q dependence of the λ-transition temperature.[5] It also might be useful to include the heat conduction through the side walls, but this is expected to be a very small effect.

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REFERENCES

1. Robert Duncan, PhD. thesis, University of California, Santa Barbara, 1988.
2. Melora Larson, PhD. thesis, University of California, Santa Barbara, 1993.

3. M. Larson, F-C Liu, and U. E. Israelsson, *Czech. J. Phys.* **46** 179 (1996).
4. W.H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes in C, 2nd edition,* (Cambridge University Press, New York, 1992).
5. R. Haussmann and V. Dohm, *Phys. Rev. Lett.* **67**, 3404 (1991); *Phys. Rev. B* **46**, 6361 (1992).